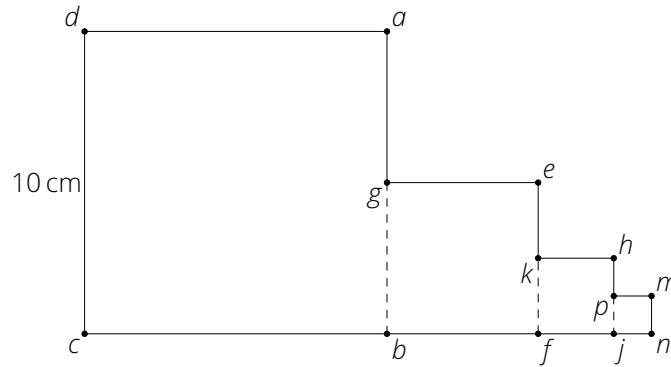# UOJMC Sample Test — Year 10

Question 1

Polly builds a complex shape out of four squares. The side length of each of her squares is half that of the previous square, the largest having a side length of 10 cm.

(a) What is the area of the square bounded by points a, b, c, and d?

(i) 50 cm²   (ii) 80 cm²   (iii) 100 cm²   (iv) 40 cm²

(b) What is the perimeter of the square bounded by points e, f, b, and g?

(i) 20 cm   (ii) 25 cm   (iii) 40 cm   (iv) 10 cm

(c) What is the perimeter of the area bounded by points a, g, e, f, c, and d?

(d) What is the distance between p and j?

(e) What is the exact area of the shape bounded by points a, g, e, k, h, p, m, n, c, and d?

Question 2

An investment scheme offers a savings rate of 9% per year, compounded annually. Suppose $10,000 is deposited in the scheme from the outset, and assume no money is taken out of the scheme for 10 years.

(a) After one year, how much *profit* has been made?

(i) $900   (ii) $1000   (iii) $850   (iv) $800

(b) After two years, how much *profit* has been made?

(i) $1714   (ii) $1831   (iii) $1881   (iv) $1664

(c) How much *profit* has been made after five years? Round your answer to 2 decimal places.

Question 3

Fred has a pot plant sitting on his window sill. Over the course of the week he measures the growth in the height of the plant in millimetres:

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
7	2	7	6	6	4	3

(a) How many millimetres did the plant grow over the course of the week?

<div align="center">(i) 30 mm   (ii) 35 mm   (iii) 38 mm   (iv) 40 mm</div>

(b) How much on average did the plant grow each day?

<div align="center">(i) 4 mm   (ii) 3 mm   (iii) 5 mm   (iv) 6 mm</div>

(c) Suppose each day over the next week the plant grew by the median amount of growth seen each day of the first week. How much would it grow?

Question 4

Tama works at a jewellery store. One day, Tama records how many of each item he sells as well as the total profits made per item. This information is stored in the following table:

Item	Number Sold	Total Profit
Bracelets	18	$270
Rings	23	$460
Necklaces	13	$169
Anklets	5	$40

Assume that for each type of jewellery, the same profit is made on every item of that type sold.

(a) For the four types of items combined, what is the total profit made during that day?

<div align="center">(i) $868   (ii) $902   (iii) $983   (iv) $939</div>

(b) Quinn buys one of each item. How much profit does Tama make out of Quinn's purchase?

(c) Jenny asks Tama to pick out one item of jewellery at random that she will then buy. What is the probability that she buys a ring? State your answer as a fraction.

Question 5

The amount of electrical power p that flows through a circuit (measured in watts or W) can be determined by the relationship $p = vi$, where v is the voltage (measured in volts or V) and i is the current (measured in amperes or A).

(a) How much power flows through a circuit if the voltage is 7V and the current is 1.5A?

(i) 3.5W   (ii) 8.75W   (iii) 10.5W   (iv) 8.5W

(b) How much voltage is required if the desired power for a circuit is 70W and the current is fixed at 5A?

(i) 350V   (ii) 14V   (iii) 700V   (iv) 7V

Batteries supply power at a fixed voltage for a period determined by their ampere-hour (Ah) rating. For example a 9 volt battery supplies 9 volts with an ampere-hour rating of 0.2 Ah, which means that drawing 1A from the battery will supply a constant 9 watts of power for 0.2 hours (after which the battery will be "flat" and no longer supply power).

Batteries may be connected in parallel to provide power for a longer period of time. When connected like this their ampere-hour ratings are added together.

(c) If a constant power supply of 12W is needed from the battery described above, for how long will the battery last?

(i) 0.2 hours   (ii) 0.3 hours   (iii) 0.15 hours   (iv) 0.1 hours

(d) What is the minimum number of batteries (as described above, in parallel) needed to supply a constant 3W for 2 hours?

Question 6

2022 is a multiple of 6.

(a) What is the smallest number larger than 2022 that is a multiple of 19?

(i) 2021   (ii) 2033   (iii) 2031   (iv) 2028

2029 does not have 6, 11, nor 13 as factors.

(b) What is the smallest number larger than 2029 that is a multiple of both 6 and 13?

(c) What is the smallest number larger than 2029 that is a multiple of both 6 and 11?

Question 7

The *triangular numbers* are as follows: $1, 3, 6, 10, 15, 21, \ldots$ The nth triangular number is the sum of all natural numbers up to including n: $1 + 2 + 3 + 4 + \cdots + n$.

(a) What is the 8th triangular number?

$$(i)\ 45 \quad (ii)\ 28 \quad (iii)\ 36 \quad (iv)\ 32$$

(b) How many triangular numbers are there under 100?

We can create another sequence of numbers, the *tetrahedral number* by adding the triangular numbers together. The first six numbers of this sequence are 1, 4, 10, 20, 35, and 56.

(d) What is the 8th tetrahedral number?

$$(i)\ 120 \quad (ii)\ 96 \quad (iii)\ 84 \quad (iv)\ 108$$

(e) How many tetrahedral numbers are there under 200?

A sequence similar to the triangular numbers is the *pyramid numbers*, which add the squares of the natural numbers together. Thus the first six pyramid number are 1, 5, 14, 30, 55, and 91.

(f) What is the 8th pyramid number?

$$(i)\ 224 \quad (ii)\ 240 \quad (iii)\ 140 \quad (iv)\ 204$$

(g) How many pyramid numbers are there under 600?
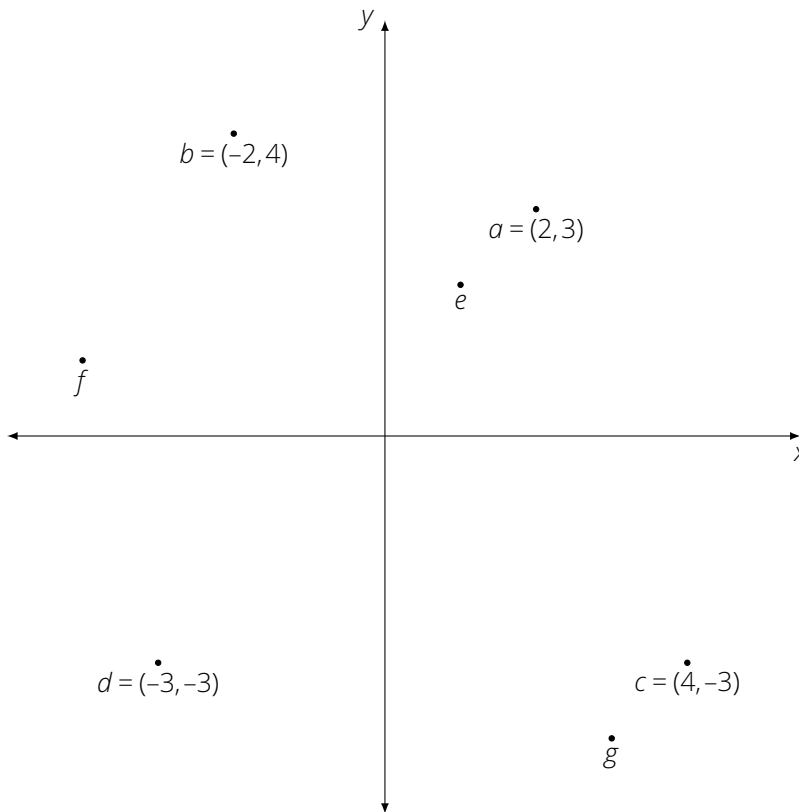
Question 8

In each part of this question provide the smallest and largest integer in your list of consecutive integers. Each correct integer earns one mark.

(a) There are *three* consecutive integers that add to 18. What are the smallest and largest of these?

(b) There are *four* consecutive integers that add to 14. What are the smallest and largest of these?

(c) There are *four* consecutive *odd* integers that add to 40. What are the smallest and largest of these?

The following diagram is relevant to both Questions 9 and 10.

Question 9

(a) In the diagram, point *e* is closest to what co-ordinate?

(i) (0, 3)   (ii) (3, -2)   (iii) (-1, 1)   (iv) (1, 2)

(b) In the diagram, point *f* is closest to what co-ordinate?

(i) (4, 0)   (ii) (-3, -2)   (iii) (-4, 1)   (iv) (-3, 2)

(c) In the diagram, point *g* is closest to what co-ordinate?

(i) (4, 3)   (ii) (-3, -3)   (iii) (-3, 4)   (iv) (3, -4)

Question 10

(a) In the diagram, which point out of *a*, *b*, *c*, or *d* is *closest* to the origin (0,0)?
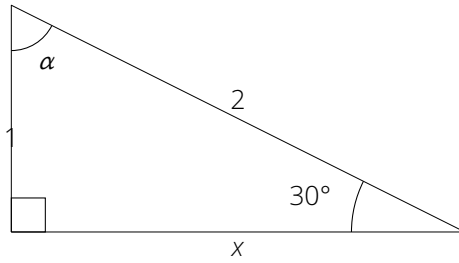
(i) *a*   (ii) *b*   (iii) *c*   (iv) *d*

(b) In the diagram, which point out of *a*, *b*, *c*, or *d* is *furthest* from the origin (0,0)?

(i) *a*   (ii) *b*   (iii) *c*   (iv) *d*

(c) Find the shortest distance from *b* to *c*. Round your answer to 2 decimal places.

Question 11

(a) In the pictured triangle, what is the size of the angle labelled α?

(i) 30°   (ii) 45°   (iii) 60°   (iv) 72°

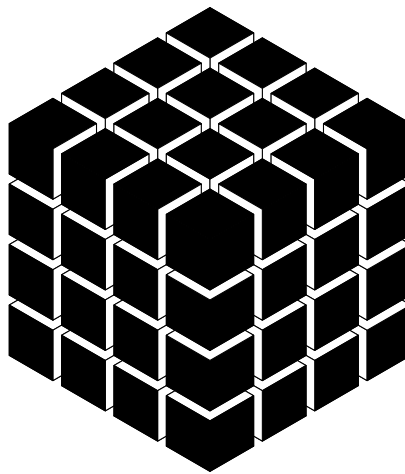(b) In the pictured triangle, what is the length of the side labelled x?

(i) 2   (ii) 3   (iii) $\sqrt{2}$ (iv) $\sqrt{3}$

(c) What is the area of the pictured triangle?

(i) $\sqrt{3}$ (ii) $\sqrt{2}$ (iii) $\frac{1}{\sqrt{3}}$ (iv) $\frac{\sqrt{3}}{2}$

Question 12

Take a cube of side 4 units, paint it black and then slice it into unit cubes as indicated in the diagram. (You may assume all interior sides of the smaller unit cubes are not black.)

(a) How many of the unit cubes are black on 3 sides?

(i) 9 cubes   (ii) 8 cubes   (iii) 7 cubes   (iv) 6 cubes

(b) How many of the unit cubes are black on 2 sides?

(c) How many of the unit cubes are black on no sides?

Question 13

In this question (a), (b), and (c) are worth 1 mark each, and (d) is worth two marks. Give your probabilities as fractions.

(a) How many letters (counting duplicates) are there in MISSISSIPPI?

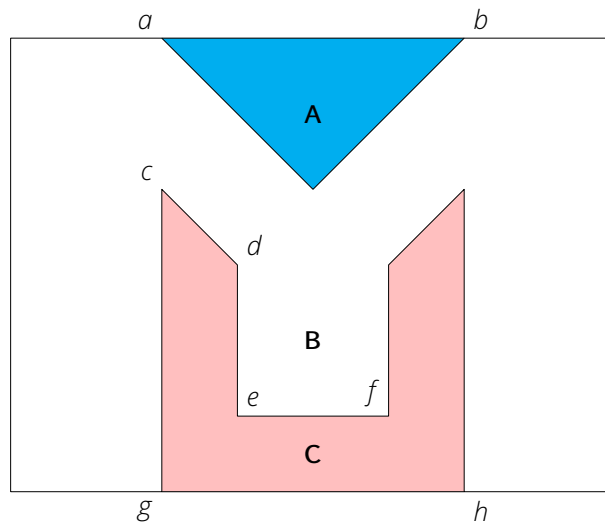Suppose that a letter is chosen randomly from the word MISSISSIPPI.

(b) What is the probability that the chosen letter is an S?

(c) What is the probability that the chosen letter is a P?

(d) What is the probability that the chosen letter is a P or an I?

Question 14

Rahul produces the following logo design for a company:

The logo (which is symmetric about its vertical axis) measures 8cm wide by 6cm high. All the angles in the logo are integer multiples of 45°. The lines $\overline{ab}$, $\overline{cg}$, and $\overline{gh}$ measure 4cm, the lines $\overline{ef}$ and $\overline{de}$ measure 2cm, and the line $\overline{cd}$ measures $\sqrt{2}$cm.

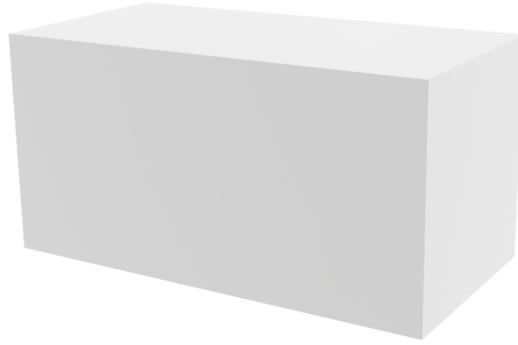(a) Which of the following is the area of the isosceles triangle shaded blue and labelled **A**?

(i) $6\,\text{cm}^2$ (ii) $8.25\,\text{cm}^2$ (iii) $4\,\text{cm}^2$ (iv) $4.5\,\text{cm}^2$

(b) What is the area of the polygon shaded pink and labelled **C**?

(c) What is the area of the polygon shaded light grey and labelled **B**?

Question 15

Consider a cuboid with a width of 9 cm, a height of 4 cm, and a depth of x cm. (A cuboid is like a cube, but not all the side lengths are the same. The pictured cuboid is **not** to scale.)

(a) If the volume of the cuboid is 252 cm^3, which of the following is the correct value for x?

$$\text{(i) 7 cm} \quad \text{(ii) 9 cm} \quad \text{(iii) 4 cm} \quad \text{(iv) 6 cm}$$

(b) What is the value of x if the surface area of the cuboid is 371 cm^2?

(c) Ignoring dimensions, if the surface area of the cuboid is equal to its volume, what is the value of x?

Question 16

A regular pack of cards has four suits, namely hearts, diamonds, spades, and clubs. Each suit consists of ten numbered cards (with the card numbered one being referred to as an 'ace'), and also cards named Jack, Queen, and King (the picture cards).

(a) What is the probability that the top card in a freshly shuffled (randomized) pack is a club?

$$\text{(i) 1/4} \quad \text{(ii) 1/10} \quad \text{(iii) 1/5} \quad \text{(iv) 1/3}$$

(b) What is the probability that the top card in a freshly shuffled pack is a Queen?

$$\text{(i) 1/4} \quad \text{(ii) 1/6} \quad \text{(iii) 1/12} \quad \text{(iv) 1/13}$$

(c) What is the probability that the top two cards in a freshly shuffled pack are both aces? Give your answer as a decimal and round your answer to 4 decimal places.

(d) What is the probability that the top four cards in a freshly shuffled pack are each one of 4, 5, 6, or 7 (in any order, more than one of each is possible)? Give your answer as a decimal and round your answer to 4 decimal places.