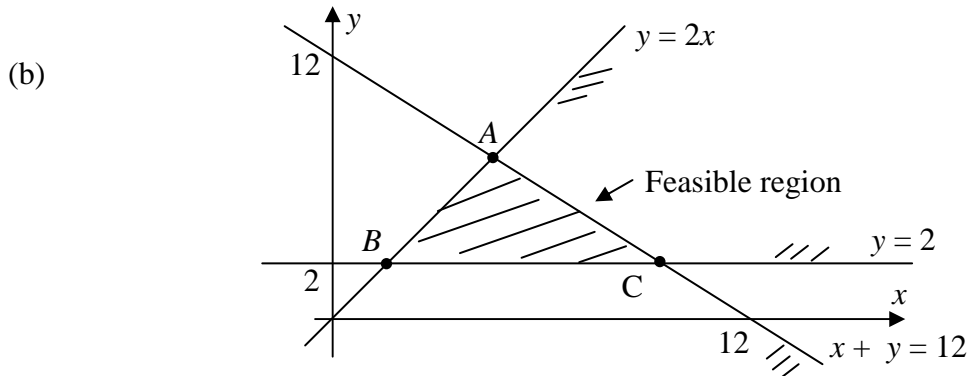


# MATH 151 Exam Answers 2011 Semester 1

1. (a)  $y = (5/4)x - 5$       (b)  $5/4 (= 1.25)$       (c)  $-5$       (d)  $-4/5$
2. (a) Let  $x =$  number of X-ploers and  $y =$  number of Y-goslows  
 (i)  $x + y = 50$       (ii)  $4x + 3y = 171$
- (b)  $x = 21, y = 29$
3. (a) (i)  $x + y \leq 12$     (ii)  $y \geq 2$       (iii)  $y \leq 2x$

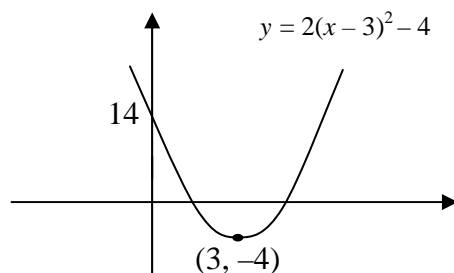


- (c)  $I = 0.1x + 0.2y$
- (d) Vertices:  $A(4, 8), B(1, 2), C(10, 2)$

	$0.1x + 0.2y$
$A(4, 8)$	$0.4 + 1.6 = 2$ Max
$B(1, 2)$	$0.1 + 0.4 = 0.5$
$C(10, 2)$	$1 + 0.4 = 1.4$

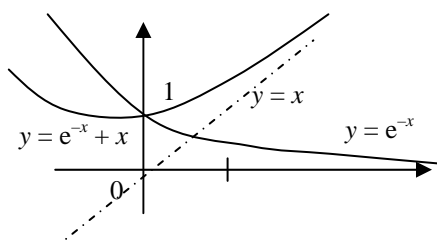
Max interest  $C = \$2000$  when  $x = 4, y = 8$ .

4. (a)  $A = 4x^2 + 30x$   
 (b)  $x = 5/2$  (discard  $-10$ )
5. (a)  $y = 2x^2 - 12x + 14 = 2[x^2 - 6x + 7]$   
 $= 2[(x-3)^2 - 3^2 + 7] = 2(x-3)^2 - 4$ .
- (b) Vertex  $(3, -4)$ ,  $y$  int = 14

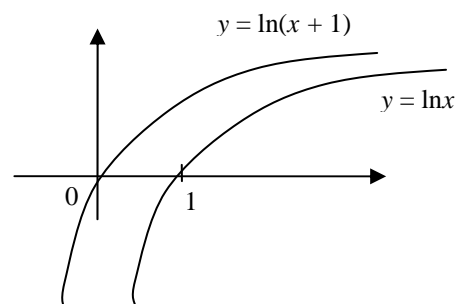


6. (a)  $f(g(x)) = 2(3x^2 + 1) - 5 = 6x^2 - 3$   
 (b)  $f^{-1}(x) = \frac{x+5}{2}$ .

7. (a)

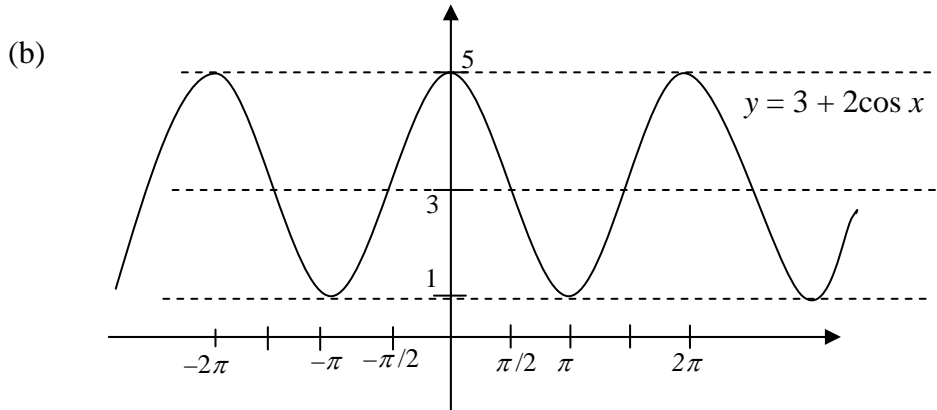


- (b)



8. (a)  $100(1.05)^8 = \$147.75$  (b)  $\$122.78$   
 (c) 14.2 six month periods or 7.1 years

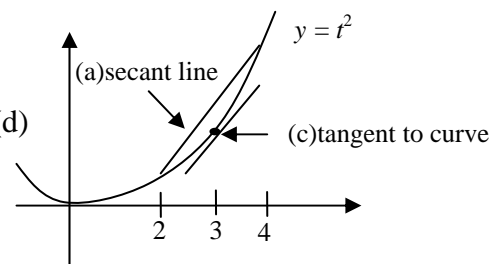
9. (a) (i)  $r = 13$  (ii)  $\sin\theta = -12/13$  (iii)  $\theta = \cos^{-1}(-12/13) = -67.4^\circ$ .



10. (a)  $A^{-1} = \frac{1}{2} \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}$  (b)  $\begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

(c)  $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 7 \\ -4 \end{bmatrix}$  so  $x = 3.5$ ,  $y = -2$

11. (a) 6 m/sec (b) 27 (c) 3 sec (d)



12. (a) 4 (b)  $y + 1 = 4(x - 1)$  so  $y = 4x - 5$

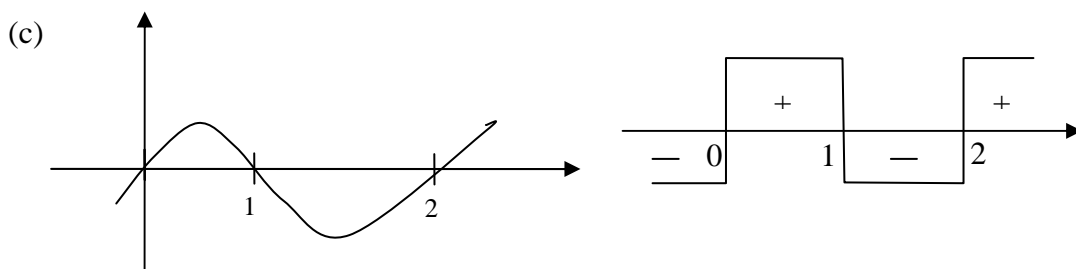
13. (a) Since  $y + 2x = 80$ ,  $A = xy = x(80 - 2x) = 80x - 2x^2$  (b)  $x = 20$ ,  $A = 800$ .

14. (a)  $\frac{d^2y}{dx^2} = 12x^2 - 12x - 72$  (b)  $-2 < x < 3$

15. (a)  $f'(x) = \frac{2}{x} - 2x = 0$  when  $x^2 = 1$ , so  $x = 1$  (b)  $y = 6x - \frac{2}{x} - 2$ ,  $y = 3x^2 < 0$ .  
 Therefore  $x = 1$  is a local max.

16. (a)  $P'(t) = 4t^3 - 12t^2 + 8t$

(b)  $P'(t) = 4t(t^2 - 3t + 2) = 4t(t - 1)(t - 2)$  so stationary values at  $t = 0, 1, 2$



(d) Share price decreasing when  $P'(t) < 0$  so  $1 < t < 2$ .

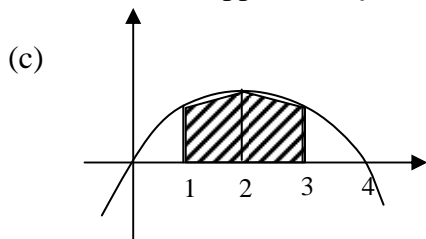
(e) From sign diagram there is a local max at  $t = 1$ ,  $P(1) = \$6$

(f)  $P(1) = 6$  local max,  $P(4) = \$69$  so absolute max = \$69 occurs at  $t = 4$ .

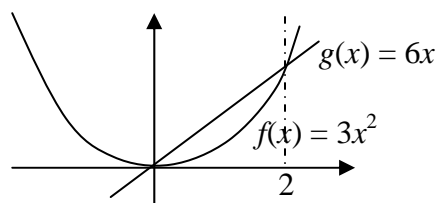
17. (a)  $(1/98)x^{98} - 2x^{3/2} + C$       (b)  $2 \ln x + 3 \sin x + C$

18. (a)  $I = \int_1^3 (4x - x^2) dx = [2x^2 - x^3/3]_1^3 = 9 - 1 \frac{2}{3} = 7 \frac{1}{3}$ .

(b) Area is approx  $\frac{1}{2} [f(1) + 2f(2) + f(3)] = 7$



19.  $f(x) = g(x) \Rightarrow x = 0$  or  $x = 2$ .



Area enclosed between the graphs =

$$\int_{-1}^3 (g(x) - f(x)) dx = \int_0^2 (6x - 3x^2) dx = [3x^2 - x^3]_0^2 = 4$$

20. (a) DE:  $\frac{dP}{dt} = kP$       Solution:  $P = Ce^{kt}$ .

(b)  $P = 12$  when  $t = 0 \Rightarrow C = 12$

$P = 24$  when  $t = 2 \Rightarrow 24 = 12 e^{2k} \Rightarrow e^{2k} = 2$

So when  $t = 6$ ,  $P = 12 e^{6k} = 12 (e^{2k})^3 = 12(2)^3 = 96$

21.  $f(x) = \sin x$ ,  $f'(x) = \cos x$ ,  $f''(x) = -\sin x$ ,  $f'''(x) = -\cos x$ ,  $f''''(x) = \sin x$

so  $f(0) = 0$ ,  $f'(0) = 1$ ,  $f''(0) = 0$ ,  $f'''(0) = -1$ ,  $f''''(0) = 0$

and therefore  $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$  (see notes page 126)