Rules of Arithmetic

If a, b, c and d are real numbers then

- $a+b=b+a$ and $ab=ba$ (commutativity)
  - $3+4=4+3$ and $(3)(4)=(4)(3)$
  - $3-4=3+(-4)=-4+3=-1$: Subtracting is the same as adding a negative number
    $3\div 4=\frac{3}{4}=(3)(1/4)=(1/4)(3)$: Dividing is the same as multiplying with a fraction (the *reciprocal* of the *divisor*)
- $a+(b+c)=(a+b)+c=a+b+c$ and $a(bc)=(ab)c=abc$ (associativity)
  - It doesn't matter which numbers you add or multiply first (if you only add or only multiply)
- $a(b+c)=(ab)+(ac)$ (distributive law)
  - $(3)(4+5)=(3)(4)+(3)(5)$ (expanding the expression)
    - $2a(3b+4c)=6ab+8ac$
  - $(4)(3)+(5)(3)=(4+5)(3)$ (factoring the expression)
    - $4a+5a=(4+5)a=9a$
    - $6b+9bc=(2+3c)(3b)$
- $(a+b)(c+d)=ac+bc+ad+bd$
  - $(a+b)^2=(a+b)(a+b)=a^2+ba+ab+b^2=a^2+2ab+b^2$
  - $(a-b)^2=a^2+(-b)a+a(-b)+(-b)^2=a^2-2ab+b^2$
  - $(a+b)(a-b)=a^2+ba-ab-b^2=a^2-b^2$
    - $(22)(18)=(20+2)(20-2)=(20)^2-(2)^2=400-4=396$
- Adding fractions and cancelling:
  - You can only combine fractions if they have the same denominator!
    - $\frac{2}{3}+\frac{3}{4}=\frac{(2)(4)}{(3)(4)}+\frac{(3)(3)}{(4)(3)}=\frac{(2)(4)+(3)(3)}{(3)(4)}=\frac{8+9}{12}=17/12$
    - $\frac{2}{3a}+\frac{3}{4b}=\frac{(2)(4b)+(3)(3a)}{12ab}=\frac{9a+8b}{12ab}$
  - You can only cancel factors if denominator and numerator are in product form!
    - $\frac{3a+6b}{3c}=\frac{(3)(a+2b)}{3c}=\frac{a+2b}{c}$
- Zero factors: if $ab=0$, then either $a=0$ or $b=0$, or both! Proof?

Exponents (Powers)

We give a quick review of exponents, but these should be practised on Mathercize if required.

For any number a and any positive integer n we define $a^n = \underbrace{a\times a\times\ldots\times a}_{\leftarrow\ n\ \text{times}\ \rightarrow}$.

a is called the *base* and n the *exponent* (or *power*).

We also define

(i) $a^0=1$	(ii) $a^{-n}=\frac{1}{a^n}$, the reciprocal of a^n.

For example: $4^3 = 4\times 4\times 4 = 64$, $\left(\frac{1}{3}\right)^2 = \frac{1}{3}\times\frac{1}{3} = \frac{1}{9}$, $(-2)^3 = (-2)\times(-2)\times(-2) = -8$,

$4^{-3} = \frac{1}{4^3} = \frac{1}{64}$, $5^0 = 1$, $7^1 = 7$.

Most scientific calculators have a button (usually labelled x^y) to calculate powers.

Three easy rules follow for integer exponents:

(i)	$a^n a^m = a^{n+m}$	(ii)	$\frac{a^n}{a^m} = a^{n-m}$	(iii)	$(a^n)^m = a^{nm}$

Mathercize practice: Math 151 Revision → Exponents → Topics