Tutorial 1 Sample Problems

These solutions are intended as a guide only. There are certainly other correct ways to do each problem. Additionally, in the solutions, some of the steps may not be spelled out in detail. If you have trouble understanding them, please see your lecturer or tutor.

Note also that you may possibly gain full marks for the assignment without writing down as much information as the solutions provide. However, you should always strive to explain clearly and succinctly what you are doing, which will involve using some words – not just bits of unrelated mathematics!

In the first 4 problems you are expected to use the algebraic approach given. However, for completeness, a more intuitive approach is also provided.

1. *Gail has a collection of seats totalling 18 in all. Some are chairs (with 4 legs) and some are bar stools (with 3 legs). There are 63 legs altogether. How many seats of each type are there?*

   All the seats have at least three legs. That accounts for 54 legs. The extra 9 legs are chair-legs, so there must be 9 chairs and so 9 stools.

   Using algebra, let S stand for the number of stools, and C for the number of chairs. ✓ The information given tells us (i) $S + C = 18$ (number of seats) and (ii) $3S + 4C = 63$ (number of legs). ✓

   Now (ii) – 3 × (i) gives: $(3S + 4C) - (3S + 3C) = 63 - 3(18) = 9$.

   That is so $C = 9$, and thus $S = 9$. ✓✓

   Of course, you could also substitute $S = 18 - C$ from (i) into (ii) and solve for C.

2. *Arnold has half as much money as Boris, and Claire has \$10 more than Arnold. Altogether they have \$100. How much money does each of them have?*

   If we take \$10 away from Claire, then together Claire and Arnold have the same amount of money as Boris and all together they have \$90. So Boris has one half of \$90, i.e. Boris has \$45. Now Arnold has half of \$45 or \$22.50 and Claire has \$10 more than Arnold, or \$32.50.

   Using algebra, let A stand for the amount of money Arnold has, B stand for the amount Boris has, and C the amount Claire has. ✓ We are told:

   $B = 2A$, $C = A + 10$, $A + B + C = 100$. ✓
   Thus substituting for B and C into the last equation we have:

   $100 = A + B + C = A + 2A + A + 10 = 4A + 10$.

   So $4A = 90$ and thus $A = \$22.50$. $B = 2A = \$45$ and $C = A + 10 = \$32.50$. ✓✓

3. *Sue is six times as old as Kim. In six years time Sue will be three times as old as Kim. How old are they now?*

We could make a table with Sue and Kim's ages now and in 6 years time. We start by guessing an age for Kim and adjusting it until we get the right relationship between their ages in 6 years time.

Now		In 6 years		
S	K	$S + 6$	$K + 6$	$S + 6 = 3(K+6)$?
12	2	18	8	×
18	3	24	9	×
24	4	30	10	✓

Using algebra, let S denote Sue's current age in years and let K denote Kim's. √ Since Sue is six times as old as Kim, we have

(1) $S = 6K$.
In six years Sue will be $S + 6$ years old, Kim will be $K + 6$ and Sue is three times as old as Sue. So we also have

(2) $S + 6 = 3(K + 6)$. √
Substituting for S from (1) into (2) and expanding the RHS we have

(3) $6K + 6 = 3K + 18$
Subtract $3K + 6$ from both sides gives

(4) $3K = 12$, so $K = 4$.
Thus Kim is 4 years old and Sue is 24. √√

4. *George bought a few textbooks at an average cost of $45 each. Then he had to buy two more which cost $100 each. The average cost of his textbooks was now $50 each. How many textbooks had he originally purchased?*

The extra $50 that each of the new textbooks cost over and above the new average of $50, was enough to raise the average cost of the old textbooks by $5 each. The two 'extra' $50's represent twenty lots of $5, so there were twenty books before, and twenty two now. Checking all this, we see the first twenty books cost $900, the two new ones bring the total to $1100, and the average price is 1100/22 = $50.

The solution by algebra is a little more complicated this time. Let N represent the number of textbooks purchased before the two new ones. √ Since the average price of these was $45, the total cost of them was 45N$. So the total cost together with the new textbooks is $(45$N$ + 200), and we now have $N + 2$ books. √ The information about the average now says:

$$\frac{45N + 200}{N + 2} = 50 \quad \checkmark \text{ so}$$

$$45N + 200 = 50N + 100$$

$$100 = 5N$$

$$20 = N \quad \checkmark$$

5. *Solve the pair of simultaneous equations* $3x + 5y = 11$
$5x - 3y = 7.$

Given (i) $3x + 5y = 11$ 3 × (i) $9x + 15y = 33$
(ii) $5x - 3y = 7$ 5 × (ii) $25x - 15y = 35$ √√

Adding $34x = 68$
so $x = 2$ √

and thus from (i) $5y = 11 - 6 = 5$ so $y = 1$. √

6 (a) *Find the slope of the line through the points*
(i) $(-2, -1)$ *and* $(4, 3)$ *(ii)* $(0, 5)$ *and* $(4, -1)$.

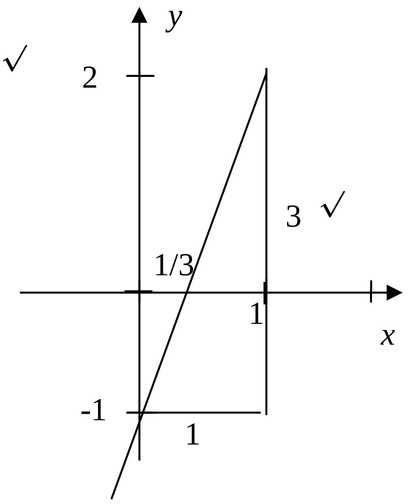
(i) Slope $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-(-1)}{4-(-2)} = \dfrac{4}{6} = \dfrac{2}{3}$ √

(ii) Slope $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-5}{4-0} = -\dfrac{6}{4} = -\dfrac{3}{2}$. √

(b) *Find the equation of the line with slope 3 and y-intercept –1. Sketch its graph, noting the x intercept.*

$m = 3$ and $c = -1$ so $y = mx + c = 3x - 1$.

When $y = 0$, $3x = 1$, so $x = 1/3$ is the x-intercept. √



(b) *Find the slope, x and y-intercepts and sketch each line below*

(i) $3y - 4x = 6$

$3y - 4x = 6 \Rightarrow y = \dfrac{4}{3}x + 2$ so has slope $\dfrac{4}{3}$ and y-intercept 2.

Now $y = 0 \Rightarrow -4x = 6 \Rightarrow x = -1.5$ is x-intercept. √

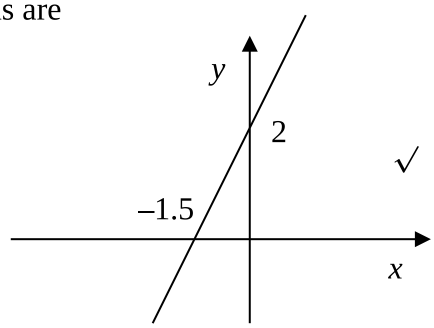
(ii) $3x + 5y = 15.$

$3x + 5y = 15 \Rightarrow y = -\dfrac{3}{5}x + 3$ so has slope $-\dfrac{3}{5}$ and y-intercept 3.

Now $y = 0 \Rightarrow 3x = 15 \Rightarrow x = 5$ is x-intercept. √

Graphs are

(i)



(ii)