Tutorial 2 Sample Problems

These solutions are intended as a guide only. There are certainly other correct ways to do each problem. Additionally, in the solutions, some of the steps may not be spelled out in detail. If you have trouble understanding them, please see your lecturer or tutor.

Note also that you may possibly gain full marks for the assignment without writing down as much information as the solutions provide. However, you should always strive to explain clearly and succinctly what you are doing, which will involve using some words – not just bits of unrelated mathematics!!

1. *Give the equations of each of the following lines:*

(i) *The line through (–3, 5) with slope 2.*

$y - 5 = 2(x - (-3))$ So, simplifying, gives $y - 5 = 2x + 6$ or $y = 2x + 11$. ✓

(ii) *The line through (2, 3) and (4, –5).*

The slope is $m = \frac{-5 - 3}{4 - 2} = \frac{-8}{2} = -4$, ✓ so the equation of the line is

$y - 3 = -4(x - 2)$ or $y = -4x + 11$. ✓

(iii) *The line through (3, 1) which is parallel to $y = -5x + 5$?*

Since our line has slope –5 and it passes through (3, 1) we have
$y - 1 = -5(x - 3)$ which gives $y = -5x + 16$ ✓

(iv) *The line through (1, 2) which is perpendicular to $y = -(1/3)x + 4$.*

The slope of the given line is $\frac{-1}{3}$. Hence the slope of a perpendicular line is 3. ✓ Thus an equation of the perpendicular line through (1, 2) is
$y - 2 = 3(x - 1)$ or $y = 3x - 1$. ✓

2. *A tank is being filled with water at a constant rate. 10 minutes after filling is started, the tank contains 13.6 litres of water. After 26 minutes the tank contains 25.6 litres of water.*

(a) *Find the slope of the line joining the two points given. Interpret (in words) what the slope is measuring.*

The straight line representing the relationship between x and y passes through $(x_1, y_1) = (10, 13.6)$ and $(x_2, y_2) = (26, 25.6)$

Thus the slope is $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{25.6 - 13.6}{26 - 10} = \dfrac{3}{4}$ or 0.75. √

The slope represents the constant rate at which the tank is filling (0.75 L/min). √

(b) *Find the equation of the line giving the volume of water y in the tank after x minutes.*

The equation of the line is $y - 13.6 = 0.75(x - 10)$ or $y = 0.75x + 6.1$. √

(c) *Find the initial volume of water in the tank.*

If $x = 0$ then $y = 6.1$. Initially the volume in the tank is 6.1 L. √

(d) *Find how long it takes to fill the tank if the maximum capacity of the tank is 80 L.*

$y = 0.75x + 6.1$

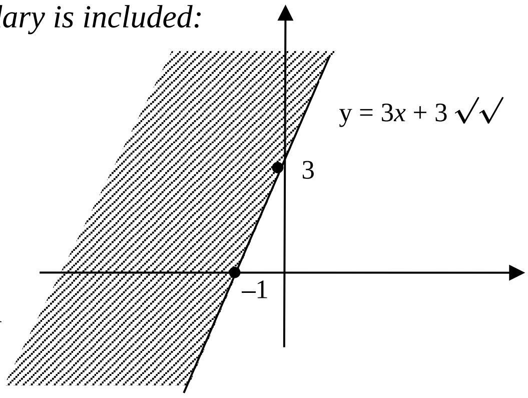
So if $y = 80$ then $80 = 0.75x + 6.1$ √

That is $73.9 = 0.75x$, and dividing both sides by 0.75 we get $x = 98.53$ √ min.

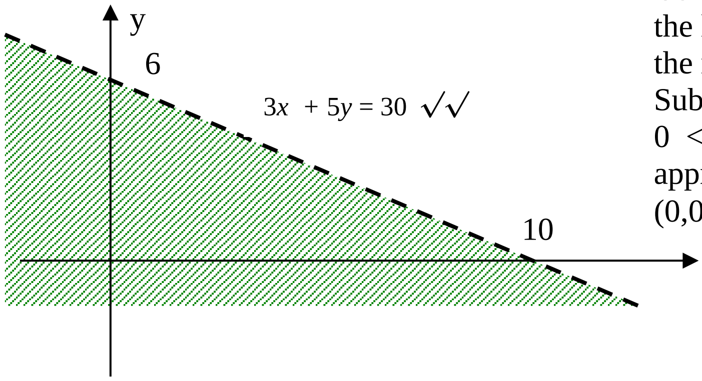
3. *On separate axes sketch the regions defined by each individual inequality below, indicating carefully whether or not the boundary is included:*

(i) $y \geq 3x + 3$

Using the intersection method gives the line shown. Testing the point (0, 0) gives $0 \geq 3$, which is false. So the appropriate half-plane is on the opposite side to (0, 0). The line is included (and so is shown as a solid line).



(ii) $3x + 5y < 30$.

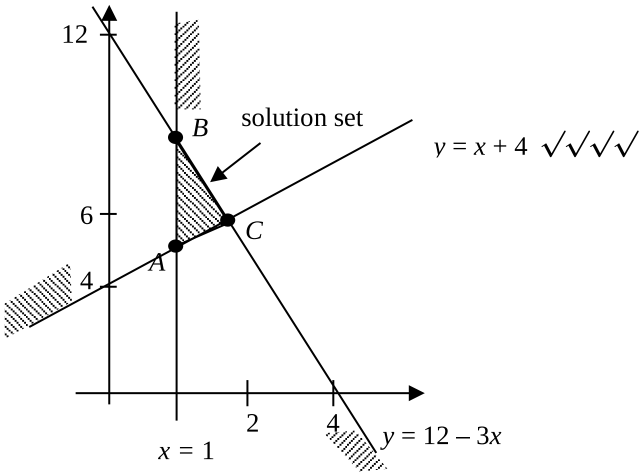


Using the intersection method gives the line shown. It is not included in the region so it is shown dotted. Substituting in the point (0, 0) gives $0 < 30$ which is true, so the appropriate half-plane contains (0,0) and is below the line.

4. (a) *Graph the solution set of the system of inequalities*

$$x \geq 1,\ y - x \geq 4,\ \ y \leq 12 - 3x.$$

Use the method given in the notes. First draw and label the corresponding line, then indicate which side of the line is required. Finally shade in the region where all the inequalities are satisfied (the solution set).



(b) *Find the vertices (corner points) of the region.*

A is the point on $y = x + 4$ where $x = 1$. So A is (1, 5).
B is the point on $y = 12 - 3x$ where $x = 1$. So B is (1, 9). √
C is the point where $x + 4 = 12 - 3x$, so $4x = 8$, that is $x = 2$, $y = 6$. √

5. *The W.E. Droppem company are considering setting up a new bungy jumping operation. They can make their bungy cords out of red strands, blue strands, or both. A red strand costs 10 cents, while a blue strand costs 15 cents. Each red strand contributes 2 units of strength, and 4 units of bounciness to the cord, while each blue strand contributes 1 unit of strength, and 4 units of bounciness to the cord. For safety reasons, a completed cord must have at least 1000 units of strength. For customer happiness, a completed cord must also have at least 2400 units of bounciness.*

(a) *Write down the constraints determined by this scenario.*

Let x be the number of red strands used. Let y be the number of blue strands used. Then the constraints are:

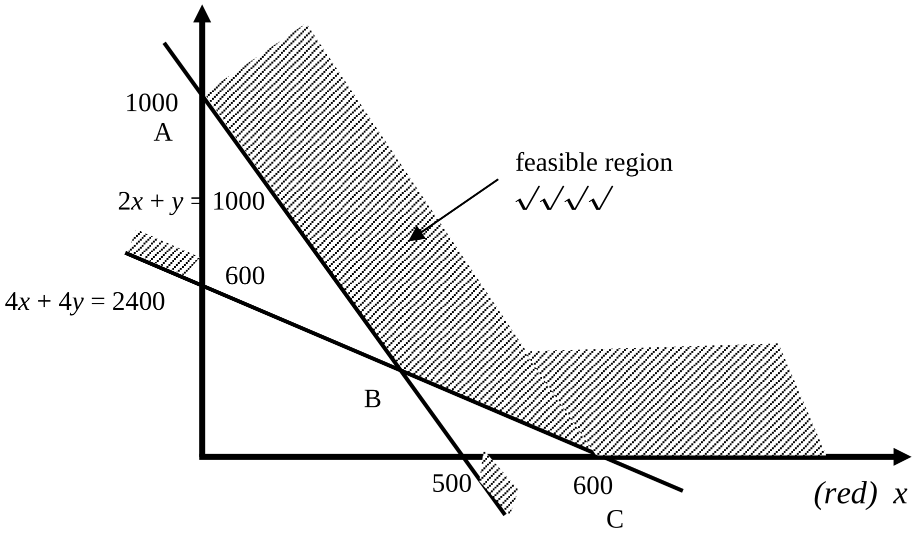
Strength $\quad 2x + y \geq 1000$ √

Bounciness $\quad 4x + 4y \geq 2400$ √

(b) *Write down the expression for the cost of a cord.*

$C = 10x + 15y$ (in cents) or $C = 0.1x + 0.15y$ (in dollars) √

(c) *Sketch the feasible region for this problem (label the axes appropriately).*



(d) *Use the vertex method to determine how many strands of each colour should be used to minimise the cost of a cord.*

B is the point where $2x + y = 1000$ meets $4x + 4y = 2400$
that is $x + y = 600$. Subtract the second from the first to get
$x = 400$, $y = 200$.
Therefore B is the point (400, 200). √
There are three points to evaluate in the Cost Function:

Point	**P = 0.1*x* + 0.15*y***	
A (0, 1000)	0 + 150 = 150	Method √√
B (400, 200)	40 + 30 = 70	
C (600, 0)	60 + 0 = 60	

The minimum cost is $60. This occurs if the company uses 600 strands of red cord and no blue cord. √