

## Tutorial 3 Sample Problems solutions

These solutions are intended as a guide only. There are certainly other correct ways to do each problem. Additionally, in the solutions, some of the steps may not be spelled out in detail. If you have trouble understanding them, please see your lecturer or tutor.

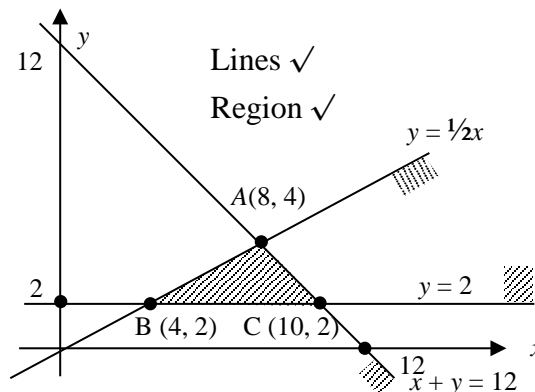
Note also that you may possibly gain full marks for the assignment without writing down as much information as the solutions provide. However, you should always strive to explain clearly and succinctly what you are doing, which will involve using some words – not just bits of unrelated mathematics!!

1. A trust invests  $x$  thousand dollars in stock  $X$  and  $y$  thousand dollars in stock  $Y$ . Regulations require that: the total investment must be no more than 12 thousand dollars; the investment in  $Y$  must be at least 2 thousand dollars; the investment in  $Y$  can be no greater than half the investment in  $X$ .

(a) The values of  $x$  and  $y$  are clearly non negative. List the other 3 constraints they satisfy.

$$x + y \leq 12, \quad y \geq 2, \quad y \leq \frac{1}{2}x. \quad \checkmark\checkmark$$

(b) Roughly sketch the feasible region, and find the 3 vertices.



Vertices  $A(8, 4)$ ,  $B(4, 2)$ ,  $C(10, 2)$ .  $\checkmark\checkmark$

(c) Suppose share  $X$  yields 5% interest and share  $Y$  yields 10% so the interest function is  $I = 0.05x + 0.1y$  (in thousands). Use the vertex method to find the maximum interest  $I$  (subject to the constraints) and give the corresponding values of  $x$  and  $y$ .

Point	Interest ( $0.05x + 0.1y$ )
$A(8, 4)$	0.8 (\$800)
$B(4, 2)$	0.4 Table✓
$C(10, 2)$	0.7

The interest is clearly a maximum at  $A$  ( $I = 0.8$  thousand or \$800) when  $x = 8$  and  $y = 4$ .  $\checkmark$

2. Factorise the following quadratics and thus solve for  $x$ .

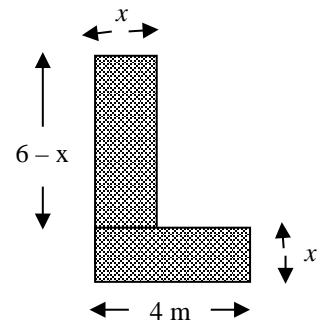
(a)  $x(x + 2) = 15$

$$x^2 + 2x - 15 = 0 \Rightarrow (x - 3)(x + 5) = 0 \checkmark \Rightarrow x = 3 \text{ or } x = -5. \checkmark$$

(b)  $5x^2 = 13x + 6$

$$5x^2 - 13x - 6 = 0 \Rightarrow (5x + 2)(x - 3) = 0 \checkmark \Rightarrow x = -\frac{2}{5} \text{ or } x = 3. \checkmark$$

3. A home handyman has sufficient materials to lay a total area of  $9 \text{ m}^2$  of concrete. He intends to make an L-shaped region whose overall dimensions are 6 m and 4 m as shown. The width of each arm is to be a uniform  $x$  metres.



(a) Write down a quadratic equation satisfied by  $x$ .

Area = Area of two rectangles shown  
 $= (6 - x)x + 4x = -x^2 + 10x \checkmark$   
 so equation is  $-x^2 + 10x = 9. \checkmark$

(b) Find the distance  $x$  by factorizing your quadratic

$$x^2 - 10x + 9 = (x - 9)(x - 1) = 0 \quad \checkmark$$

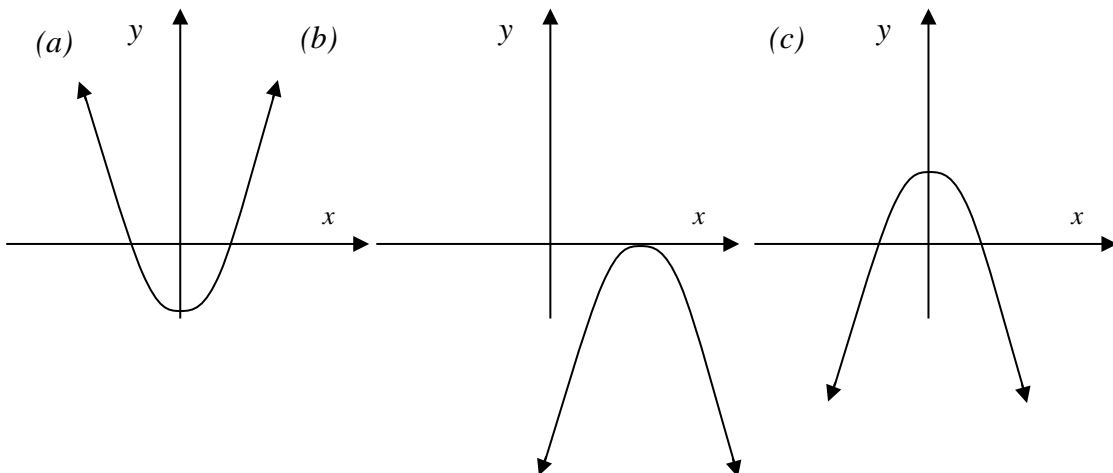
so  $x = 9$  (not possible) or  $x = 1. \checkmark$

(4) Write down the equation which best describes the given parabola. You must choose your answer for each part from the following list:

$$\begin{aligned} y &= -x^2 + 2 \\ y &= (2 - x)^2 \\ y &= -(x - 2)^2 \end{aligned}$$

$$\begin{aligned} y &= (x - 2)^2 \\ y &= (2 + x)^2 \\ y &= -(2 + x)^2 \end{aligned}$$

$$\begin{aligned} y &= 2 + x^2 \\ y &= x^2 - 2 \\ y &= -(x + 2)^2 \end{aligned}$$



- (a) The diagram shows  $y = x^2$  shifted down by 2. Therefore  $y = x^2 - 2$ . ✓
- (b) The diagram shows  $y = x^2$  shifted to the right by 2 and the sign changed. Therefore  $y = -(x-2)^2$ . ✓
- (c) This parabola opens downwards so the coefficient of  $x^2$  is negative. The diagram shows  $y = -x^2$  shifted up by 2. Therefore  $y = -x^2 + 2$ . ✓

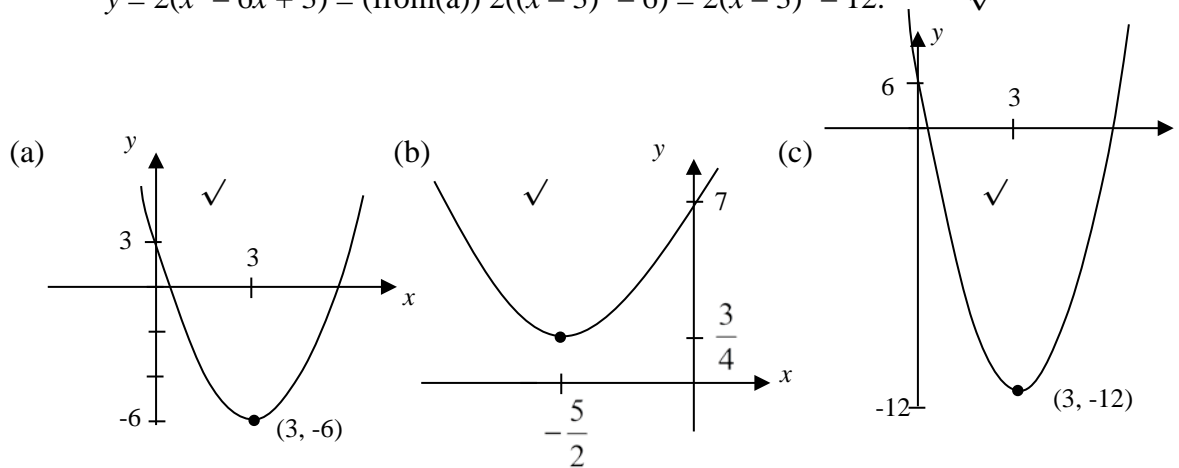
(5) Complete the square on each of the quadratic functions below and then give a rough sketch of the graph of each noting the vertex and the y-intercept.

(Note: (a) should help you to do (c)).

(a)  $y = x^2 - 6x + 3$   
 $y = x^2 - 6x + 3 = (x-3)^2 - 3^2 + 3 = (x-3)^2 - 6$  ✓

(b)  $y = x^2 + 5x + 7$   
 $y = x^2 + 5x + 7 = (x + \frac{5}{2})^2 - (\frac{5}{2})^2 + 7 = (x + \frac{5}{2})^2 + \frac{3}{4}$  ✓

(c)  $y = 2x^2 - 12x + 6$ .  
 $y = 2(x^2 - 6x + 3) = (\text{from(a)}) 2((x-3)^2 - 6) = 2(x-3)^2 - 12$ . ✓



(6) For the following, write down a quadratic equation and use the quadratic formula to solve it:

- (a) John is 29 years older than Peter. The product of their ages is 1272. How old is John?

If  $x$  represents John's age then  $x(x-29) = 1272$ ,

that is  $x^2 - 29x - 1272 = 0$ . Here  $a = 1$ ,  $b = -29$ ,  $c = -1272$ . ✓

$$\begin{aligned}
 x &= \frac{29 \pm \sqrt{(-29)^2 + 4(1272)}}{2} = \frac{29 \pm \sqrt{5929}}{2} \quad \checkmark \\
 &= \frac{29 \pm 77}{2} = \frac{106}{2} = 53 \quad \checkmark \quad \text{(or } \frac{-48}{2} = -24 \text{ which doesn't make sense).}
 \end{aligned}$$

(b) *A triangle has a base which is 2 metres longer than its (perpendicular) height. It has an area of 10 m<sup>2</sup>. Find the height of the triangle (to 2 dp).*

If the height is  $x$  metres then the area is  $\frac{1}{2}x(x+2) = 10$ , that is  $x^2 + 2x - 20 = 0$ .

Here  $a = 1$ ,  $b = 2$ ,  $c = -20$   $\checkmark$

$$\begin{aligned}
 x &= \frac{-2 \pm \sqrt{2^2 + 4(20)}}{2} = \frac{-2 \pm \sqrt{84}}{2} \quad \checkmark \\
 &= \frac{-2 \pm 9.16}{2} = \frac{7.16}{2} = 3.58 \quad \checkmark \quad \text{or } -\frac{11.56}{2} \text{ (not valid here).}
 \end{aligned}$$