# Tutorial 5 Sample Problems solutions

These solutions are intended as a guide only. There are certainly other correct ways to do each problem. Additionally, in the solutions, some of the steps may not be spelled out in detail. If you have trouble understanding them, please see your lecturer or tutor.

Note also that you may possibly gain full marks for the assignment without writing down as much information as the solutions provide. However, you should always strive to explain clearly and succinctly what you are doing, which will involve using some words – not just bits of unrelated mathematics!!

1. *By first finding the $x$ and $y$ intercepts roughly sketch the following cubics*

   (a) $y = (2x + 5)(2 - x)(x + 1)$.

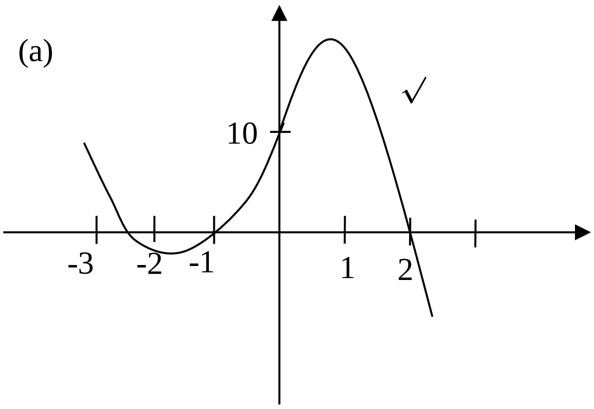
   $y = 0 \Rightarrow (2x + 5)(2 - x)(x + 1) = 0 \Rightarrow$ either $2x + 5 = 0$, so $x = -5/2$, or $2 - x = 0$, so $x = 2$, or $x + 1 = 0$, so $x = -1$.

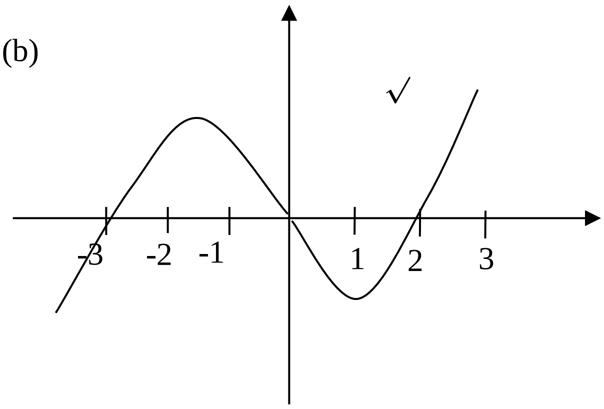
   Therefore $x$-intercepts are $-5/2, -1$ and $2$.
   $x = 0 \Rightarrow y = (5)(2)(1) = 10$, so $y$-intercept is $10$. √√

   Since $y = -2x^3 + \ldots$ ($a < 0$), curve has same general shape as $y = -2x^3$.

   (b) $y = x^3 + x^2 - 6x$

   $y = x^3 + x^2 - 6x = x(x^2 + x - 6) = x(x - 2)(x + 3)$, so $y = 0$
   $\Rightarrow x(x - 2)(x + 3) = 0 \Rightarrow$ either $x = 0$, or $x - 2 = 0$, so $x = 2$ or $x + 3 = 0$, so $x = -3$. Therefore $x$-intercepts are $-3, 0$ and $2$.
   $x = 0 \Rightarrow y = 0$ so $y$-intercept is $0$. √√
   Since $y = x^3 + \ldots$ ($a, > 0$), curve has same general shape as $y = x^3$.




2. *Scrooge McFloogle invests his entire savings of \$2,000 in the VDIC (Very Dodgy Investment Company). If they pay an interest rate of 16% per annum how much will his investment be worth in 10 years if interest is compounded*

(Throughout $P = 2000$, $I = 16\%$ per annum.)

*(a)* *annually*

If compounded annually $i = 0.16$ and $n = 10$, so $R = 2000\,(1.16)^{10} =$ \$8822.87 ✓

*(b)* *quarterly*

If compounded quarterly, $i = \dfrac{0.16}{4} = 0.04$, $n = 4 \times 10 = 40$, so

$R = 2000(1.04)^{40} = \$9602.04.$ ✓

*(c)* *monthly*

If compounded monthly, $i = \dfrac{0.16}{12} = 0.01\dot{3}$, $n = 12 \times 10 = 120$, so

$R = 2000(1.01\dot{3})^{120} = \$9801.88$ ✓

*(d)* *continuously.*

If compounded continuously, $I = 0.16$, $t = 10$, so

$R = Pe^{It} = 2000e^{1.6} = \$9906.06$ ✓

3. *The Mathemaniacal Advisory Department (MAD) owns a property which is currently valued at \$50,000. In each case below use the compound interest formula and give your answer to the nearest dollar.*

*(a)* *One advisor suggests that the rate of inflation for the property will be 4% per annum over the next ten years. Estimate the value of the property in ten years time .*

$P = 50{,}000$, $i = 0.04$, $n = 10$ so estimated property value after 10 years

$= 50{,}000\,(1.04)^{10} = \$74{,}012.$ ✓

*(b)* *Over the past ten years the rate of inflation on the property was 8% per annum. Calculate the cost of the property when MAD bought it ten years ago.*

Suppose the property was worth an amount $P$, $n = 10$ years ago with $i = 0.08$ then

$50{,}000 = P(1.08)^{10} \Rightarrow \quad P = \dfrac{50{,}000}{(1.08)^{10}} = \$23{,}159.$ ✓

*(b)* *If a second advisor suggests inflation on the property will be 5% per annum for the foreseeable future. Determine the estimated number of years (from now) for the property to double in value.*

Here $i = 0.5$ and need to determine $n$ such that $100{,}000 = 50{,}000\,(1.05)^n \Rightarrow$ $2 = (1.05)^n$ so taking logs,

$\log 2 = \log(1.05)^n = n \log 1.05$ ✓ so $n = \dfrac{\log 2}{\log 1.05} = \dfrac{0.3010}{0.02119} = 14.2$ years. ✓

4. *Solve for x*

(a) *$\log_2 x = -1$*

By definition $x = 2^{-1} = 0.5$ ✓

(b) *$\log_x 729 = 6$.*

$\log_x 729 = 6$ means $x^6 = 729 \Rightarrow x = \sqrt[6]{729} = 3$ . ✓

(Check $\log_3 729 = \log_3 3^6 = 6$.)

(c) *$\log x - \log(x-1) = \log 4$*

$\log(x/(x-1)) = \log 4$ means $x/(x-1) = 4$ so $x = 4x - 4$. That is $x = 4/3$. ✓✓

5. *The concentration of a drug in a patient's bloodstream is C milligrams per milliliter after t hours, where $C = ka^{-t}$. For drug A, k = 2 and a = 1.2. For drug B, k = 3 and a = 1.4.*

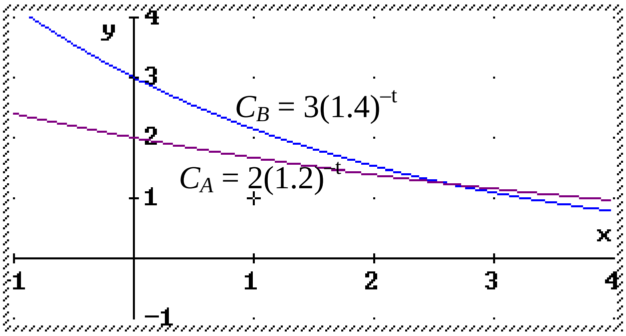
(a) *Which drug will have the greater concentration after (i) 2 hours (ii) 3 hours?*

Given $C_A = 2(1.2)^{-t}$ and $C_B = 3(1.4)^{-t}$.

(i) At $t = 2$, $C_A = 2(1.2)^{-2} = 1.389$ and $C_B = 3(1.4)^{-2} = 1.533$. ✓
So B has the greatest concentration.

(ii) At $t = 3$, $C_A = 2(1.2)^{-3} = 1.157$ and $C_B = 3(1.4)^{-3} = 1.093$ ✓
So A has the greatest concentration.

(b) *Give a (rough) sketch on the same axes of the graphs of C against t for both drugs, for $0 \le t \le 4$.*




✓✓

(c) *After what period of time would the concentration levels be the same for both drugs?*

$C_A = C_B \Rightarrow 2(1.2)^{-t} = 3(1.4)^{-t}$ ✓ $\Rightarrow \frac{2}{3} = (1.2)^t (1.4)^{-t} = \left(\frac{1.2}{1.4}\right)^t$ ✓

$\Rightarrow$ (taking logs amd rearranging) $t = \log\left(\frac{2}{3}\right)/\log\left(\frac{1.2}{1.4}\right) = 2.63$. ✓

6. *The population $P$ of New Zealand in millions is projected to grow according to the formula $P = Ce^{kt}$, where $t$ is the time in years from the present.*

*a. If at present ($t = 0$) $P = 4$, what is $C$?*

$t = 0 \Rightarrow 4 = P = Ce^0 = C1 = C$, so $C = 4$. ✓

*b. If in 5 years, $P = 4.2$, what is $k$?*

$t = 5 \Rightarrow 4.2 = P = 4e^{5k} \Rightarrow e^{5k} = \dfrac{4.2}{4} = 1.05$ ✓

$\Rightarrow$ (taking logs to base $e$) $ln(e^{5k}) = ln(1.05) \Rightarrow 5k = ln(1.05)$

$\Rightarrow k = \dfrac{1}{5} ln(1.05) = .00976$ (3*sf*). ✓

*c. What is the projected population in 10 years?*

$t = 10 \Rightarrow P = 4e^{10k} = 4e^{0.0976} \approx 4.41$ million ✓

[Alternatively $P = 4e^{10k} = 4(e^{5k})^2 = 4(1.05)^2 = 4.41$ (exactly).]